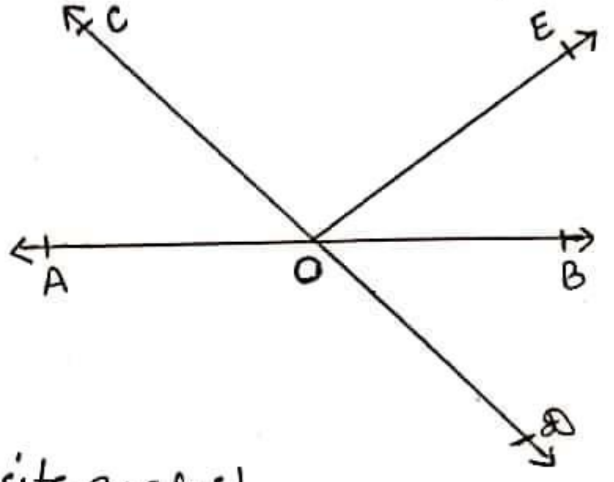
Lines and Angles

Q1 In the figure lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70^\circ$ and $\angle BOD = 40^\circ$. find $\angle BOE$ and reflex $\angle COE$



Sol: $\longrightarrow$ Lines AB and CD intersect at O

$\angle AOC + \angle BOE = 70^\circ$ (Given) $\longrightarrow$ (i)

$\angle BOD = 40^\circ$ (Given)

Since $\angle AOC = \angle BOD$ (vertically opposite angles)

$\therefore \angle AOC = \angle BOD = 40^\circ$

and $40^\circ + \angle BOE = 70^\circ$

$\Rightarrow \angle BOE = 70^\circ - 40^\circ = 30^\circ$

Also $\angle AOC + \angle BOE + \angle COE = 180^\circ$ ($\because$ AOB is a straight line)

$\Rightarrow 70^\circ + \angle COE = 180^\circ$ from (i)

$\Rightarrow \angle COE = 180^\circ - 70^\circ$

$\Rightarrow \angle COE = 110^\circ$

Reflex $\angle COE = 360^\circ - 110 = 250^\circ$

Hence $\boxed{\angle BOE = 30^\circ}$ and $\boxed{\text{reflex } \angle COE} \boxed{= 250^\circ}$ Ans

Q2 $\Rightarrow$ In the figure, lines XY and MN intersect at O If $\angle POY = 90^\circ$ and $a : b = 2 : 3$ find c

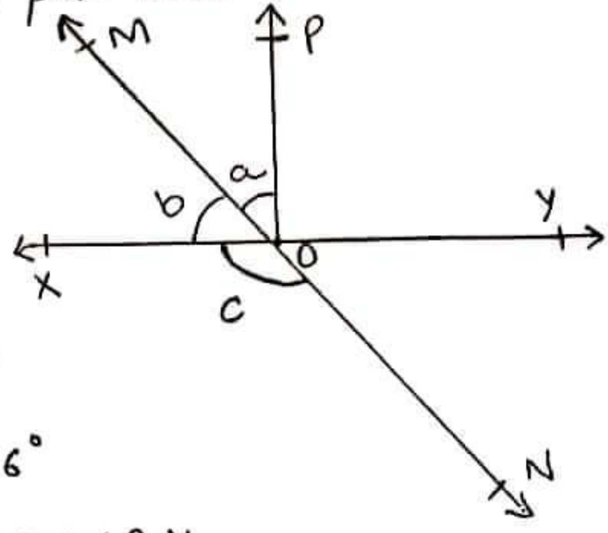
Sol: $\rightarrow$ In the figure Line XY and MN intersect at O. ~~If~~ $\angle POY = 90^\circ$

Also given $a : b = 2 : 3$

Let $a = 2x$, $b = 3x$

Since $\angle XOM + \angle POM + \angle POY = 180^\circ$

(Linear pair axiom)



$\Rightarrow 3x + 2x + 90^\circ = 180^\circ$

$\Rightarrow 5x = 180 - 90^\circ$

$\Rightarrow x = \frac{90^\circ}{5} = 18^\circ$

$\therefore \angle XOM = b = 3x = 3 \times 18^\circ = 54^\circ$

and $\angle POM = a = 2x = 2 \times 18^\circ = 36^\circ$

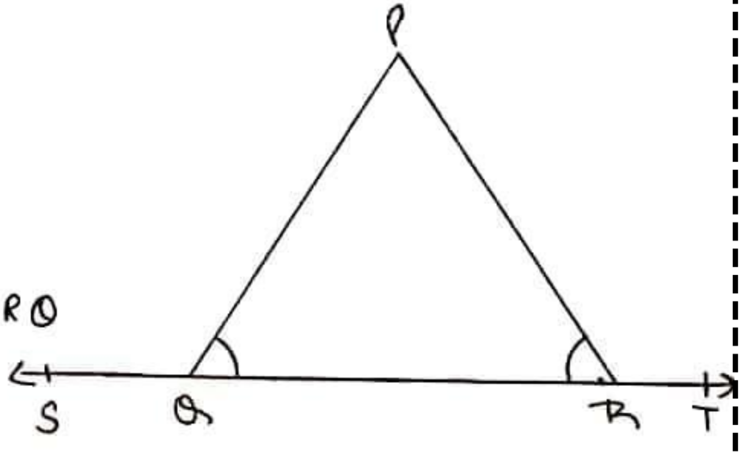
Now $\angle XON = c = \angle MOY = \angle POM + \angle POY$

(vertically opp angle)

$= 36^\circ + 90^\circ = 126^\circ$

Hence $\boxed{c = 126^\circ}$ Ans

Q3 In the figure $\angle PQR = \angle PRQ$ then prove that $\angle PQS = \angle PRT$



Sol Since $\angle PQR = \angle PRQ$

$180^\circ - \angle PQR = 180^\circ - \angle PRQ$

$\Rightarrow \angle PQS = \angle PRT$

Hence proved

Q4 In the figure if $x + y = w + z$ then prove that AOB is a line.

Sol:→ Given $x + y = w + z$

Since $x+y+z+w = 360^\circ$

$\Rightarrow x+y+x+y = 360^\circ$

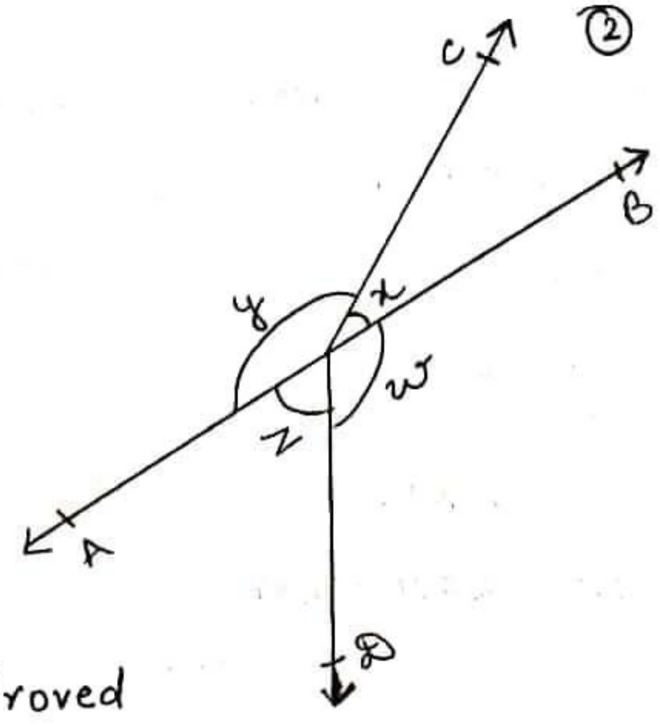
$\Rightarrow 2x+2y = 360^\circ$

$\Rightarrow 2(x+y) = 360^\circ$

$\Rightarrow x+y = \frac{360^\circ}{2}$

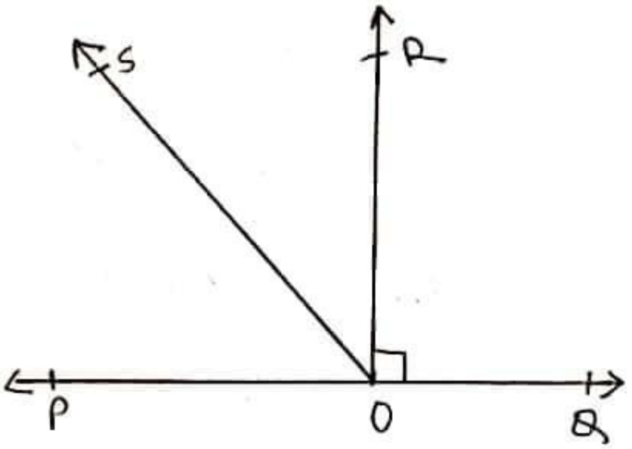
$\Rightarrow x+y = 180^\circ$

$\therefore$ AOB is a line Proved



Q5 :→ In the figure POQ is a line Ray OR is ⊥ar to line PQ. OS is another ray lying between ray OP and OR. Prove that

$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$



Sol :→ $\angle ROS = \angle ROP - \angle POS$ — (i)

and $\angle ROS = \angle QOS - \angle QOR$ — (ii)

Adding (i) & (ii) we get

$\angle ROS + \angle ROS = \angle QOS - \angle QOR + \angle ROP - \angle POS$

$\Rightarrow 2\angle ROS = \angle QOS - \cancel{90^\circ} + \cancel{90^\circ} - \angle POS$ [$\because \angle QOR = \angle ROP = 90^\circ$]

$\Rightarrow 2\angle ROS = \angle QOS - \angle POS$

$\Rightarrow \angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$

Hence proved //

Q6 $\Rightarrow$ It is given that $\angle XYZ = 64^\circ$ and XY is produced to point P. Draw a figure from the given information. If ray YO bisects $\angle ZYP$ find $\angle XYO$ and reflex $\angle OYP$

Sol: $\rightarrow$ from figure

$\angle XYZ = 64^\circ$ (Given)

Now $\angle ZYP + \angle XYZ = 180^\circ$

(Linear pair axiom)

$\Rightarrow \angle ZYP + 64^\circ = 180^\circ$

$\Rightarrow \angle ZYP = 180^\circ - 64^\circ = 116^\circ$

Also given that ray YO bisect $\angle ZYP$

But $\angle ZYP = \angle OYP + \angle OYZ = 116$

$\Rightarrow \angle OYP = \angle OYZ = \frac{116}{2} = 58^\circ$

Also $\angle XYO = \angle XYZ + \angle OYZ$

$\Rightarrow \angle XYO = 64^\circ + 58^\circ = 122^\circ$

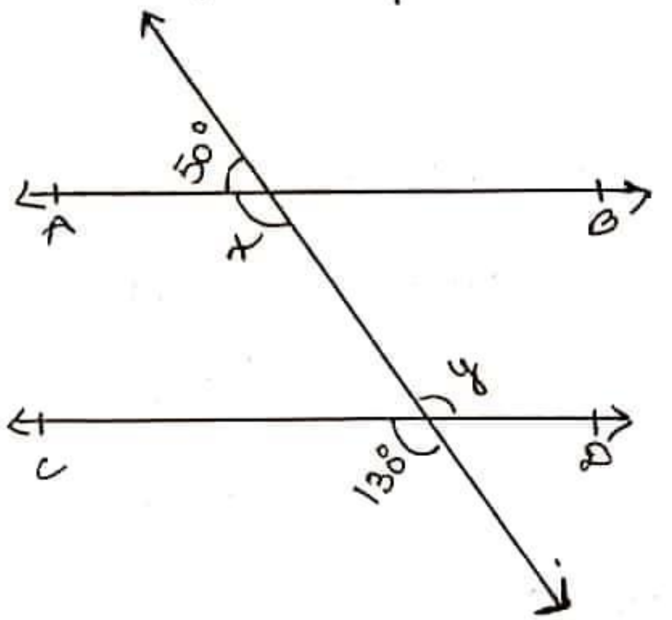
and reflex $\angle OYP = 360^\circ - \angle OYP = 360^\circ - 58 = 302^\circ$

$(\because \angle OYP = 58^\circ)$

Hence $\angle XYO = 122^\circ$ and reflex $\angle OYP = 302^\circ$ Ans.

Exercise 6.2

Q1 :→ In the figure find the value of x and y then show that $AB \parallel CD$



Sol :→ In the given figure a tranversal intersect two lines AB and CD Such that

$x + 50° = 180°$

$\Rightarrow x = 180° - 50°$

$\Rightarrow x = 130°$ —(i)

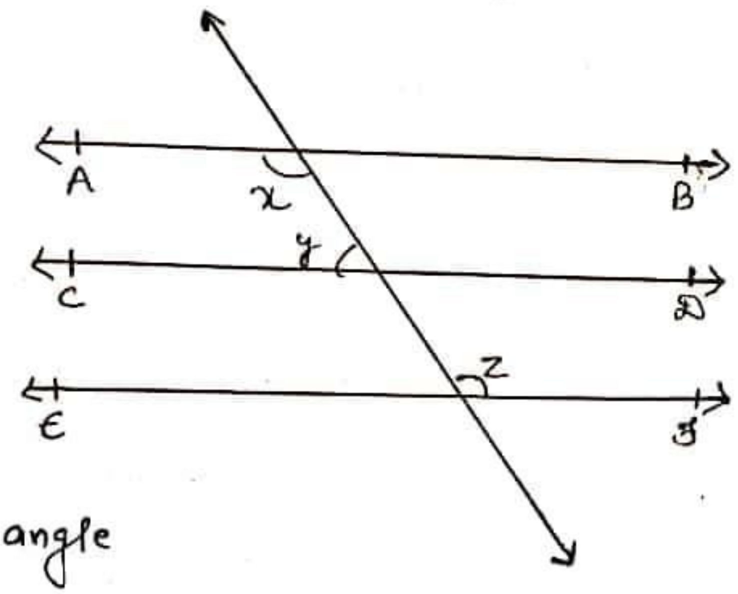
Also $y = 130°$ (vertically opposite angles) —(ii)

from (i) and (ii)

$x = y = 130°$ (Alternate angles)

$\therefore AB \parallel CD$ (Converse of alternate angles axiom)

Q2 :→ In the figure, If $AB \parallel CD$, $CD \parallel EF$ and $y : z = 3 : 7$, find x



Sol :→ In the given figure

$AB \parallel CD$, $CD \parallel EF$

and $y : z = 3 : 7$

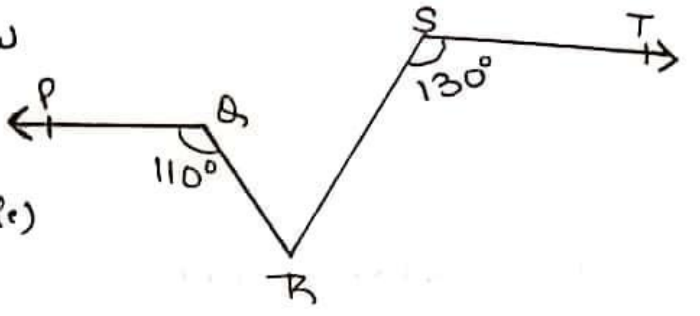
Let $y = 3a$, $z = 7a$

$\angle DHI = y$ vertically opp angle

$\angle DHI + \angle FIH = 180°$ (Interior angle on same side of transversal)

Q4 :→ In the figure, If PQ ∥ ST, $\angle PQR = 110°$ and $\angle RST = 130°$ find $\angle QRS$

Sol :→ Extend PQ to X and draw LM ∥ ST through R



$\angle TSX = \angle QXS$ (Alternate angle)

$\Rightarrow \angle QXS = 130°$

$\angle QXS + \angle RXQ = 180°$

(Linear pair axiom)

$\Rightarrow \angle RXQ = 180° - 130° = 50°$ $\because \angle QXS = 130°$

Also $\angle PQR + \angle LRQ = 180°$

(Linear pair axiom)

$\Rightarrow 110° + \angle LRQ = 180°$

$\Rightarrow \angle LRQ = 180° - 110° = 70°$

Now $\angle QRL + \angle QRS + \angle SRM = 180°$

$\Rightarrow 70° + \angle QRS + 50° = 180°$

$\Rightarrow 120° + \angle QRS = 180°$

$\Rightarrow \angle QRS = 180° - 120° = 60°$

$\Rightarrow \boxed{\angle QRS = 60°}$

Q5 ⟹ In figure If AB ∥ CD, $\angle APQ = 50°$ and $\angle PRD = 127°$ find x and y

Sol :→ In the given figure AB ∥ CD, $\angle APQ = 50°$, $\angle PRD = 127°$

$\Rightarrow y + z = 180°$

$\Rightarrow 3a + 7a = 180°$

$\Rightarrow 10a = 180° \Rightarrow a = 18°$

$\therefore y = 3a = 3 \times 18° = 54°$

and $z = 18 \times 7 = 126°$

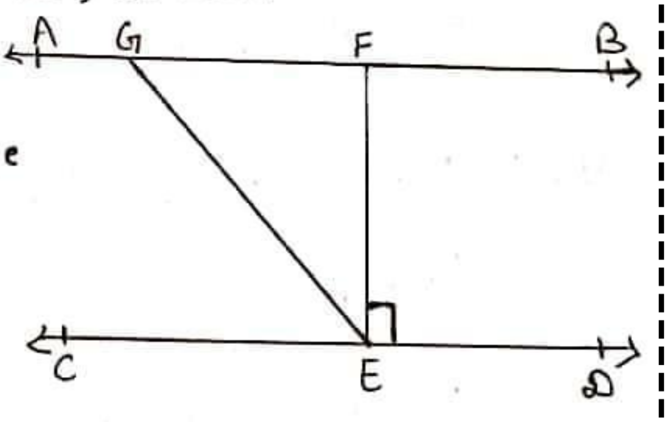
Also $x + y = 180°$

$x + 54° = 180° \Rightarrow x = 180° - 54° = 126°$

Hence $x = 126°$ Ans.

Q3: → In the figure, If $AB \parallel CD$, $EF \perp CD$ and $\angle GED = 126°$. Find $\angle AGE$, $\angle GEF$ and $\angle FGE$

Sol: → In the given figure $AB \parallel CD$, $EF \perp CD$ and $\angle GED = 126°$



$\angle AGE = \angle GED$ (Alternate angle)

$\therefore \angle AGE = 126°$

Now $\angle GEF = \angle GED - \angle DEF$

$= 126° - 90° = 36°$ ($\because \angle DEF = 90°$)

Also $\angle AGE + \angle FGE = 180°$ (Linear pair axiom)

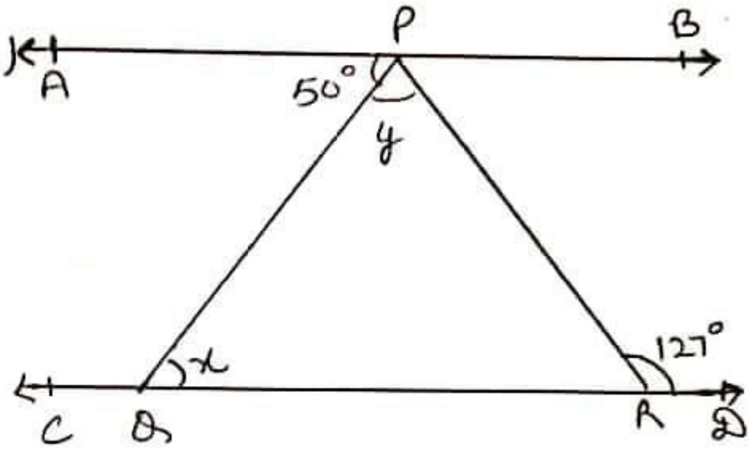
$\Rightarrow 126° + \angle FGE = 180°$

$\Rightarrow \angle FGE = 180° - 126°$

$\Rightarrow \angle FGE = 54°$

$\angle APR = \angle PRD = 127°$

(Alternate angles)



$\therefore$ $50° + y = 127°$

$\Rightarrow y = 127° - 50° = 77°$

$\Rightarrow \boxed{y = 77°}$

Also $\angle PRD + \angle PRQ = 180°$

(Linear Pair axiom)

$\Rightarrow 127° + \angle PRQ = 180°$

$\Rightarrow \angle PRQ = 180° - 127° = 53°$

Now in ΔPQR

$x + y + \angle PRQ = 180°$

$\Rightarrow 77 + x + 53° = 180°$

$\Rightarrow x + 130° = 180°$

$\Rightarrow x = 180° - 130° = 50°$

$\Rightarrow \boxed{x = 50°}$

Q6 In the figure ---- Prove that $AB \parallel CD$

Sol At Point B, draw $BE \perp PQ$ and at point C draw $CF \perp RS$

$\angle 1 = \angle 2$ —(i)

(Acc to law of reflection)

$\angle 3 = \angle 4$ (Same reason) —(ii)

Also $\angle 2 = \angle 3$ (Alternate angle) —(iii)

From (i) (ii) + (iii)

$\angle 1 = \angle 4$

$\Rightarrow 2\angle 1 = 2\angle 4$

$\Rightarrow \angle 1 + \angle 1 = \angle 4 + \angle 4$

$\Rightarrow \angle 1 + \angle 2 = \angle 3 + \angle 4$

$\Rightarrow \angle BCD = \angle ABC$

Hence $AB \parallel CD$ $\because$ (Alternate angles are equal)